REALISM IN MATHEMATICS

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CLARENDON PAPERBACKS
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For
Dick and Steve
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PREFACE

The philosophy of mathematics is a borderline discipline, of fundamental importance to both mathematics and philosophy. Despite this, one finds surprisingly little co-operation between philosophers and mathematicians engaged in its pursuit; more often, widespread disregard and misunderstanding are broken only by alarming pockets of outright antagonism. (The glib and dismissive formalism of many mathematicians is offset by the arrogance of those philosophers who suppose they can know what mathematical objects are without knowing what mathematics says they are.) This might not matter much in another age, but it does today, when the most pressing foundational problems are unlikely to be answered without a concerted co-operative effort. I have tried in this book to do justice to the concerns of both parties, to present the background, the issues, the proposed solutions on a neutral ground where the two sides can meet for productive debate.

For this reason, I’ve aimed for a presentation accessible to both non-philosophical mathematicians and non-mathematical philosophers and, if I’ve succeeded, students and interested amateurs should also be served. As far as I can judge, very little philosophical training or background is presupposed here. Mathematical prerequisites are more difficult to avoid, owing to the relentlessly cumulative nature of the discipline, but I’ve tried to keep them to a minimum. Some familiarity with the calculus and its foundations would be helpful, though surely not necessary. And the relevant set theoretic concepts are referenced to Enderton’s excellent introductory textbook (see his (1977)), for the benefit of those innocent of that subject.

The central theme of the book is the delineation and defence of a version of realism in mathematics called ‘set theoretic realism’. In this, my deep and obvious debt is to the writings of the great mathematical realists of our day: Kurt Gödel, W. V. O. Quine, and
Hilary Putnam (in the early 1970s). More personally, I have learned most from John Burgess, Paul Benacerraf, Hartry Field, and Tony Martin. After these, it would be impossible to mention everyone, but I can’t overlook the forceful criticisms of Charles Chihara, the insightful comments of Anil Gupta, and the generous correspondence, assistance, and advice of Philip Kitcher and Michael Resnik. Most recently, Burgess, Field, Lila Luce, Colin McLarty, Martin, Alan Nelson, Resnik, Stewart Shapiro, Mark Wilson, and Peter Woodruff have all done me the service of reading and reacting to drafts of various parts of the manuscript. (Naturally, the remaining errors and oversights should be charged to my shortcomings rather than to their negligence.) And finally, what I owe to my long-time companion Steve Maddy is too complex and varied to be summarized here. I am grateful to all these people and offer my heartfelt thanks. Also to Angela Blackburn and Frances Morphy of Oxford University Press.

Much of this book is based on a series of articles (Maddy 1980, 1981, 1984a, 1988a,b, forthcoming a, b) the preparation of which was supported, at various times, by the American Association of University Women, the University of Notre Dame, the National Endowment for the Humanities, the National Science Foundation, and the University of Illinois at Chicago. The original publishers kindly granted advance permission to reproduce material from these pieces; in the end, only parts of (forthcoming a) (in chapter 5, sections 1 and 2) and (forthcoming b) (in chapter 1, section 4) actually survived, so I am particularly obliged to Kluwer Academic Publishers and the Association for Symbolic Logic. Preparation of the final draft was supported by National Science Foundation Grant DIR-8807103, a University of California President’s Research Fellowship in the Humanities, and the University of California at Irvine. The help of all these institutions is hereby gratefully acknowledged.

Finally, I feel compelled to add a personal note on sexist language. Some years ago, when I first introduced the ideas behind set theory realism, constructions like ‘the set theoretic realist thinks his entities . . .’ struck me as amusing, but since then I’ve discovered that some readers and editors are legitimately disapproving of this usage. Of the many alternatives available, I’ve chosen one that does the least violence to the standard rhythm, that is, the use of ‘she’
and ‘her’ in place of ‘he’ and ‘his’ in neutral contexts. Some might find this just as politically incorrect as the automatic use of the masculine, but I sincerely doubt that phrasing like ‘when the mathematician proves a theorem, she . . .’ makes anyone tend to forget that some mathematicians are men. So I’ll stick with this policy. To those who find it distracting, I apologize; this is not, after all, a political treatise. At least you have my reasons.

P.M.

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